10.3 Curve Sketching Summary
Local and Global Max/Min

Entry Task: Consider

$$f(x) = x^3 - \frac{9}{2}x^2 - 12x + 10$$

1. Find all critical values.
2. Plug the critical values into the 2nd derivative. What can you conclude?
3. Draw the 1st and 2nd deriv. analysis number lines.

$\boxed{1}$ $f'(x) = 3x^2 - 9x - 12 \overset{?}{=} 0$

$3(x^2 - 3x - 4) \overset{?}{=} 0$

$3(x-4)(x+1) = 0$

OR USE QUAD FORMULA

$\boxed{x = -1 \text{ or } x = 4}$

$\boxed{2}$ $f''(x) = 6x - 9$

AT $x = -1$, $f''(-1) = 6(-1) - 9 = -15$

SO $f'(-1) = 0$ HORIZ. TANGENT, $f''(-1) < 0$ CONCAVE DOWN $\Big\}$ $\boxed{\text{LOCAL MAX!}}$

AT $x = 4$, $f''(4) = 6(4) - 9 = 15$

SO $f'(4) = 0$ HORIZ. TANGENT, $f''(4) > 0$ CONCAVE UP $\Big\}$ $\boxed{\text{LOCAL MIN!}}$

$\boxed{3}$

f	$\nearrow$		$\searrow$		$\nearrow$
f'	$+$	-1	$-$	4	$+$

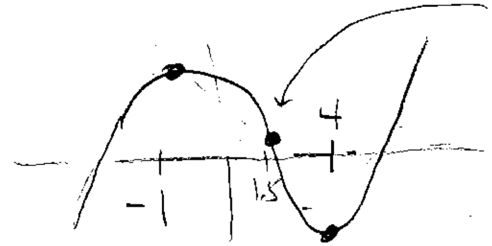
$f'(x) = 3(x-4)(x+1)$

$f''(x) = 6x - 9 \overset{?}{=} 0 \Rightarrow 6x = 9$

$\Rightarrow x = 9/6 = 3/2 = 1.5$

f	$\frown$		$\smile$
f''	$-$	1.5	$+$

POINT OF INFLECTION



Summary

For any question about "**increasing, decreasing, local max/min**", identify the function in question, $y = f(x)$, then:

Step 1: Solve $f'(x) = 0$

Step 2:

(option 1): 1st Deriv. Test

Draw 1st deriv. analysis number line.

Make appropriate conclusions.

(option 2): 2nd Deriv. Test

Plug critical numbers into 2nd deriv.

- $f'(a) = 0$, $f''(a) > 0$ $\Rightarrow$local min $\cup$
- $f'(a) = 0$, $f''(a) < 0$ $\Rightarrow$local max $\cap$

For any question about "**concave up/down or inflection points**", identify the function in question, $y = f(x)$, then:

Step 1: Solve $f''(x) = 0$

Step 2:

Draw 2st deriv. analysis number line.

Make appropriate conclusions.

Global Max/Min:

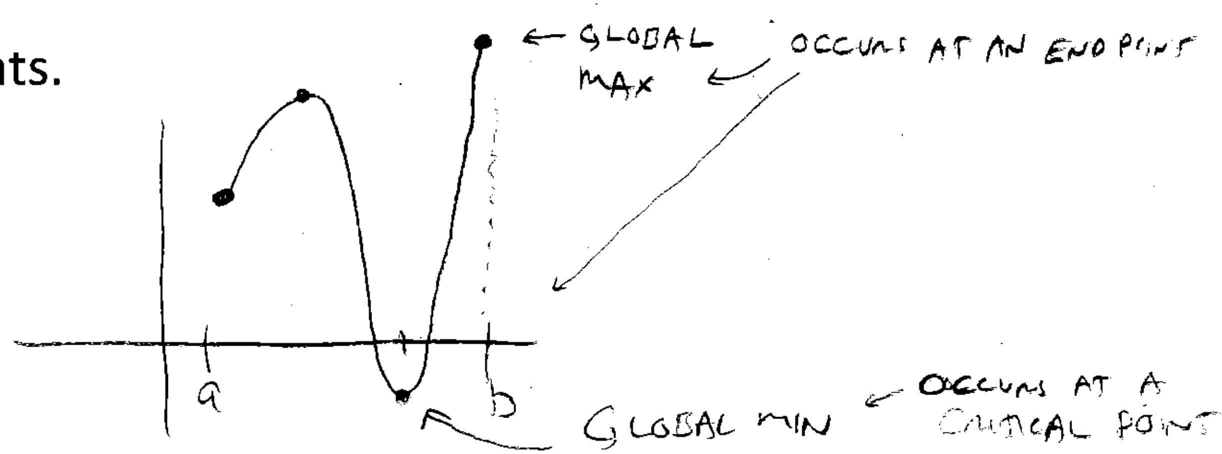
Given $y = f(x)$ and an interval $a \leq x \leq b$

The ***global maximum*** (or *absolute max*) of $f(x)$ on the interval is the highest overall y-value on that interval.

The ***global minimum*** (or *absolute min*) of $f(x)$ on the interval is the highest overall y-value on that interval

Key (awesome) Observation
(Extreme Value Thm)
The global max/min can only occur at:

a. critical values
   OR
b. endpoints.



For any question about "**global max/min**", identify the function and interval in question, $y = f(x)$ and $a \leq x \leq b$ then:

Step 1: Solve $f'(x) = 0$

Step 2:

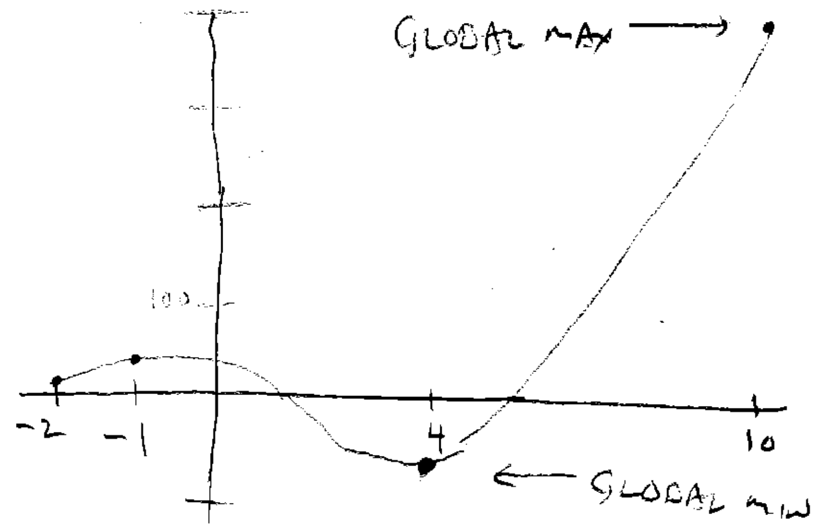
a. Plug the critical values into the original function.
b. Plug the endpoints into the original function.

At the end of step 2:
The biggest output is the global max.
The smallest output is the global min.

Example (same function from entry task):

On the interval $-2 \le x \le 10$, find the global max and min of

$$f(x) = x^3 - \frac{9}{2}x^2 - 12x + 10$$



$f'(x) = 3x^2 - 9x - 12 \overset{?}{=} 0$ already did (see page 1)

$x = -1, \quad x = 4$

$f(-1) = (-1)^3 - \frac{9}{2}(-1)^2 - 12(-1) + 10 = 16.5$

$f(4) = (4)^3 - \frac{9}{2}(4)^2 - 12(4) + 10 = -46$ ← GLOBAL MIN

$f(-2) = (-2)^3 - \frac{9}{2}(-2)^2 - 12(-2) + 10 = 8$

$f(10) = (10)^3 - \frac{9}{2}(10)^2 - 12(10) + 10 = 440$ ← GLOBAL MAX

Example (from HW 10.3/10):

The total revenue (in thousand dollars) for selling q thousand Framits is given by

$$TR(q) = \frac{1}{6}q^4 - \frac{31}{6}q^3 + 55q^2 + 200q$$

Part (c) Find the global max and global min of marginal revenue over the interval q = 0 to q = 12.

$$MR(q) = \frac{4}{6}q^3 - \frac{31}{6}3q^2 + 110q + 200$$

$$MR(q) = \frac{2}{3}q^3 - \frac{31}{2}q^2 + 110q + 200$$

← QUESTION IS ABOUT MR!!!

$$MR'(q) = 2q^2 - 31q + 110 \stackrel{?}{=} 0$$

$$q = \frac{31 \pm \sqrt{(-31)^2 - 4(2)(110)}}{2(2)} = \frac{31 \pm \sqrt{81}}{4}$$

$$= \frac{31 \pm 9}{4} = \rightarrow \begin{array}{l} 40/4 = 10 \\ \text{or} \\ 22/4 = 11/2 = 5.5 \end{array}$$

$$MR(5.5) = \frac{2}{3}(5.5)^3 - \frac{31}{2}(5.5)^2 + 110(5.5) + 200 = 447.041\overline{6}$$

$$MR(10) = \frac{2}{3}(10)^3 - \frac{31}{2}(10)^2 + 110(10) + 200 = 416.\overline{6}$$

$$MR(0) = \frac{2}{3}(0)^3 - \frac{31}{2}(0)^2 + 110(0) + 200 = 200$$

$$MR(12) = \frac{2}{3}(12)^3 - \frac{31}{2}(12)^2 + 110(12) + 200 = 440$$

GLOBAL MAX $= 447.041\overline{6}$

GLOBAL MIN $= 200$

Example: (like the last problem in HW)

Given $g(x) = \frac{1}{4}x^2 - 4x + 25$ and

$$S(x) = \frac{g(x)}{x}$$

If x is between 1 and 20, what is the smallest possible value of S(x)?

$$S(x) = \frac{\frac{1}{4}x^2 - 4x + 25}{x} = \frac{\frac{1}{4}x^2}{x} - \frac{4x}{x} + \frac{25}{x}$$

$$S(x) = \frac{1}{4}x - 4 + 25x^{-1}$$

$$S'(x) = \frac{1}{4} - 25x^{-2} \stackrel{?}{=} 0$$

$$4x^2\left(\frac{1}{4} - \frac{25}{x^2} \stackrel{?}{=} 0\right)$$

$$x^2 - 100 \stackrel{?}{=} 0$$

ONLY ONE BETWEEN 1 AND 20

$$\Rightarrow x^2 = 100 \Rightarrow x = \pm 10$$

$$S(1) = \frac{1}{4}(1) - 4 + \frac{25}{(1)} = 21.25$$

$$S(10) = \frac{1}{4}(10) - 4 + \frac{25}{(10)} = 1$$

$$S(20) = \frac{1}{4}(20) - 4 + \frac{25}{(20)} = 2.25$$

$$\boxed{\text{GLOBAL MIN} = 1}$$

Example: (like problems 5-9 of HW)

Given the monthly average cost and price for producing and selling q items:

$$AC(q) = \frac{36000}{q} + 100 + q$$

$$p = 1700$$

If production is limited to 400 items per month, what quantity maximizes profit?

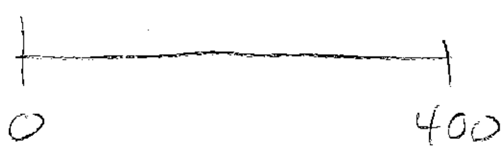
$TC(q) = q \cdot AC(q) = 36000 + 100q + q^2$

$TR(q) = p \cdot q = 1700q$

$P(q) = TR(q) - TC(q)$
$= (1700q) - (36000 + 100q + q^2)$

$P(q) = 1600q - 36000 - q^2$

WHAT IS GLOBAL MAX IF $0 \le q \le 400$?



$P'(q) = 1600 - 2q \overset{?}{=} 0$

$1600 = 2q$

$800 = q$

↑ OUTSIDE OF INTERVAL!!!

$P(0) = 1600(0) - 36000 - (0)^2 = -36000$

$P(400) = 1600(400) - 36000 - (400)^2$
$= \$444,000$ ← MAX PROFIT

$\boxed{q = 400}$ FOR $0 \le q \le 400$

NOTE: IF PRODUCTION COULD BE HIGHER, THEN MAX PROFIT WOULD OCCUR AT 800

$P(800) = \$604,000$